Chapter 6

Probability Distributions

Outline

- 6-1 Introduction
- 6-2 Probability Distributions
- 6-3 Mean, Variance, and Expectation
- 6-4 The Binomial Distribution

Objectives

- Construct a probability distribution for a random variable.
- Find the mean, variance, and expected value for a discrete random variable.
- Find the exact probability for X successes in n trials of a binomial experiment.

6-4

Objectives

- Find the mean, variance, and standard deviation for the variable of a binomial distribution.

6-2 Probability Distributions

- A variable is defined as a characteristic or attribute that can assume different values.
- A variable whose values are determined by chance is called a random variable.

6-2 Probability Distributions

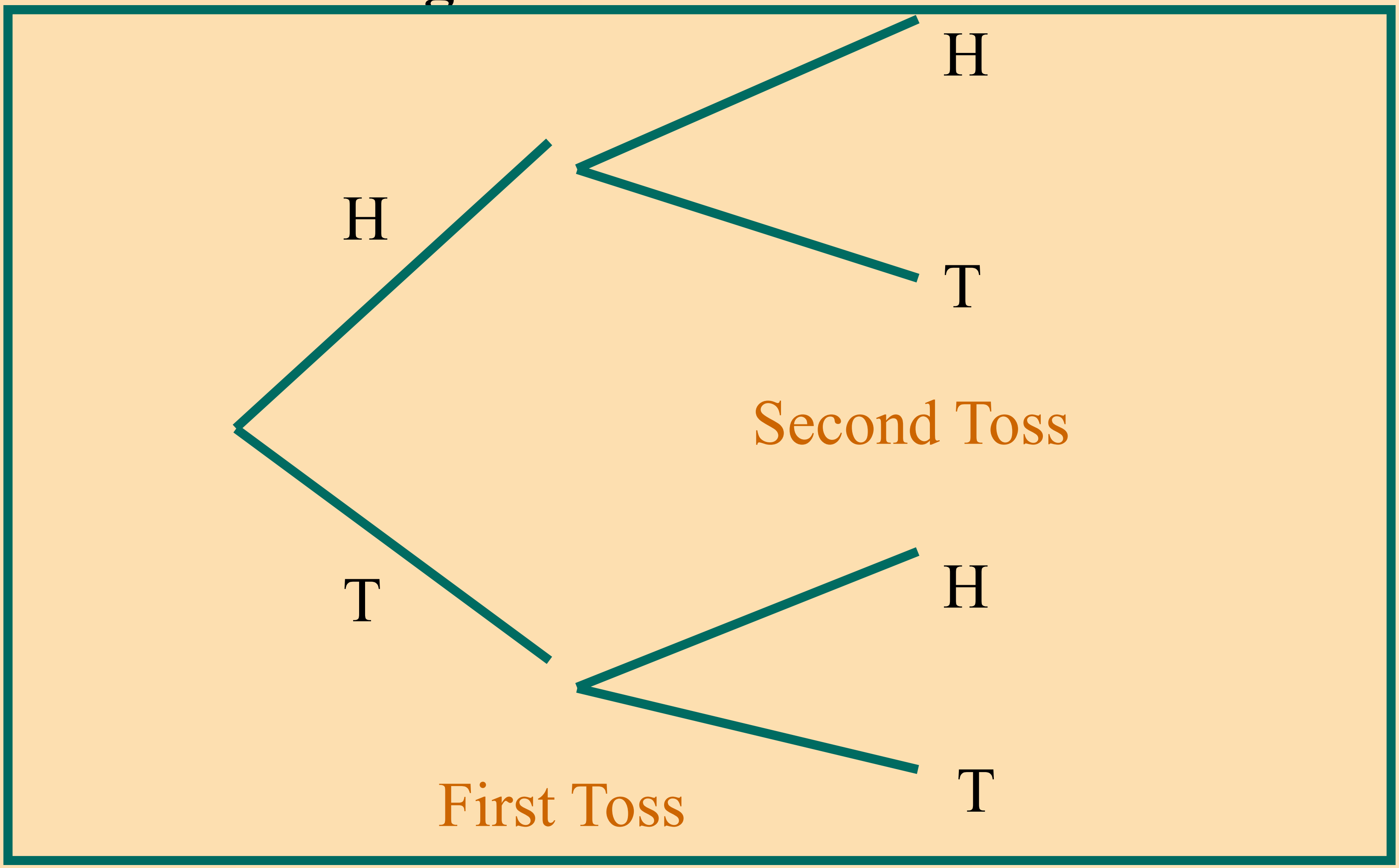
- If a variable can assume only a specific number of values, such as the outcomes for the roll of a die or the outcomes for the toss of a coin, then the variable is called a discrete variable.
- Discrete variables have values that can be counted.

6-2 Probability Distributions

- If a variable can assume all values in the interval between two given values then the variable is called a continuous variable. Example - temperature between 68^0 to 78^0.
- Continuous random variables are obtained from data that can be measured rather than counted.

6-8

6-2 Probability Distributions - Tossing Two Coins

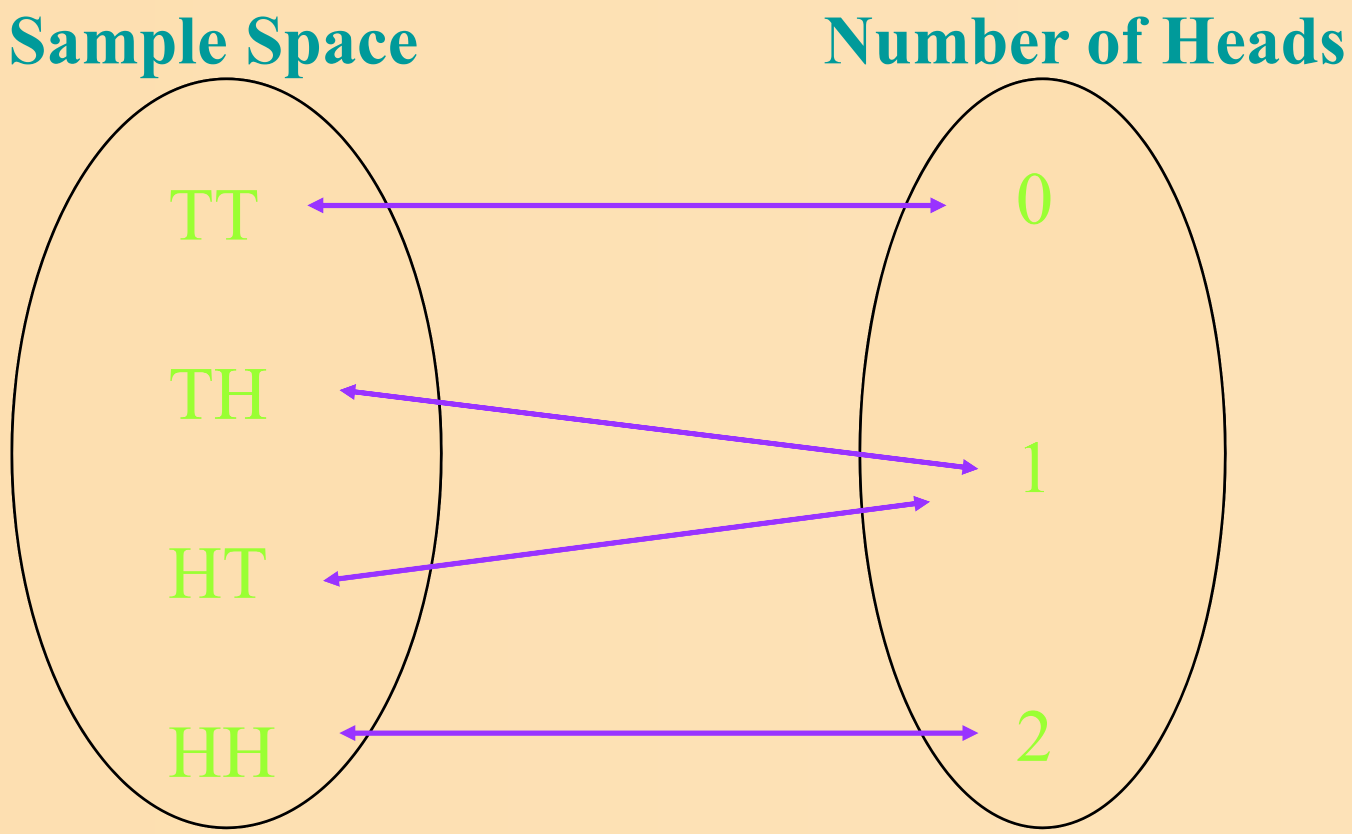


6-2 Probability Distributions - Tossing Two Coins

- From the tree diagram, the sample space will be represented by HH, HT, TH, TT.
- If X is the random variable for the number of heads, then X assumes the value 0, 1, or 2.

6-10

6-2 Probability Distributions - Tossing Two Coins



6-2 Probability Distributions - Tossing Two Coins

OUTCOME X	PROBABILITY P(X)
0	1/4
1	2/4
2	1/4

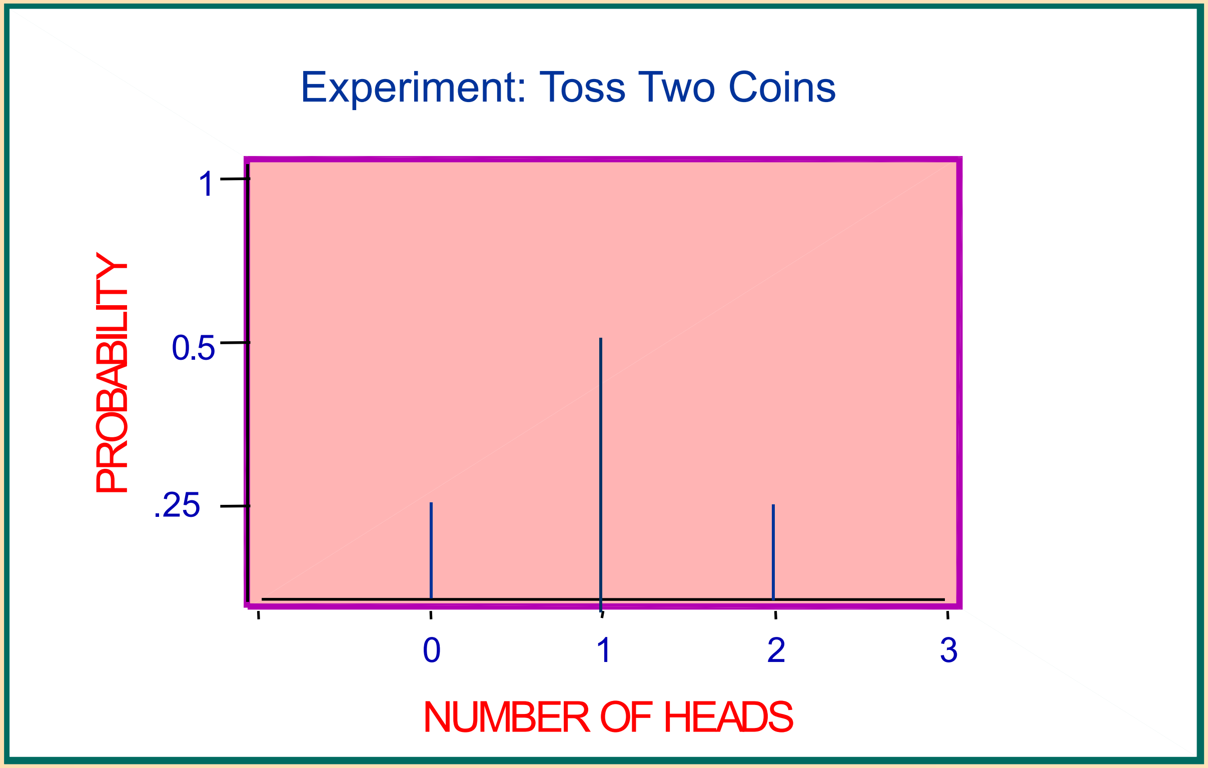
6-12

6-2 Probability Distributions

- A probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values. The probabilities are determined theoretically or by observation.

6-13

6-2 Probability Distributions -- Graphical Representation



6-3 Mean, Variance, and Expectation for Discrete Variable

The mean of the random variable of a probability distribution is

$$\mu = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + \ \dots \ + X_n \cdot P(X_n)$$
$$= \sum X \cdot P(X)$$

where $X_1, X_2, \dots, X_n$ *are the outcomes and* $P(X_1), P(X_2), \ \dots \ , P(X_n)$ *are the corresponding probabilities.*

6-15

6-3 Mean for Discrete Variable - Example

- Find the mean of the number of spots that appear when a die is tossed. The probability distribution is given below.

X	*1*	*2*	*3*	*4*	*5*	*6*
P(X)	*1/6*	*1/6*	*1/6*	*1/6*	*1/6*	*1/6*

6-3 Mean for Discrete Variable - Example

$$
\begin{aligned}
\mu &= \sum X \cdot P(X) \\
&= 1\cdot(1/6) + 2\cdot(1/6) + 3\cdot(1/6) + 4\cdot(1/6) \\
&\quad + 5\cdot(1/6) + 6\cdot(1/6) \\
&= 21/6 = 3.5
\end{aligned}
$$

That is, when a die is tossed many times, the theoretical mean will be 3.5.

6-17

6-3 Mean for Discrete Variable - Example

- In a family with two children, find the mean number of children who will be girls. The probability distribution is given below.

X	*0*	*1*	*2*
P(X)	*1/4*	*1/2*	*1/4*

6-3 Mean for Discrete Variable - Example

$$\begin{aligned}\mu &= \sum X \cdot P(X) \\ &= 0 \cdot (1/4) + 1 \cdot (1/2) + 2 \cdot (1/4) \\ &= 1.\end{aligned}$$

That is, the average number of girls in a two-child family is 1.

6-3 Formula for the Variance of a Probability Distribution

- The variance of a probability distribution is found by multiplying the square of each outcome by its corresponding probability, summing these products, and subtracting the square of the mean.

6-3 Formula for the Variance of a Probability Distribution

The formula for the variance of a probability distribution is

$$\sigma^2 = \sum\left[X^2 \cdot P(X)\right] - \mu^2.$$

The standard deviation of a probability distribution is

$$\sigma = \sqrt{\sigma^2}.$$

6-3 Variance of a Probability Distribution - Example

- The probability that 0, 1, 2, 3, or 4 people will be placed on hold when they call a radio talk show with four phone lines is shown in the distribution below. Find the variance and standard deviation for the data.

6-3 Variance of a Probability Distribution - Example

X	0	1	2	3	4
P(X)	0.18	0.34	0.23	0.21	0.04

6-3 Variance of a Probability Distribution - Example

X	$P(X)$	$X \cdot P(X)$	$X^2 \cdot P(X)$
0	0.18	0	0
1	0.34	0.34	0.34
2	0.23	0.46	0.92
3	0.21	0.63	1.89
4	0.04	0.16	0.64
		$\mu = 1.59$	$\Sigma X^2 \cdot P(X) = 3.79$

$\sigma^2 =$
$3.79 - 1.59^2$
$= 1.26$

6-3 Variance of a Probability Distribution - Example

- Now, $\mu = (0)(0.18) + (1)(0.34) + (2)(0.23) + (3)(0.21) + (4)(0.04) = 1.59$.
- $\Sigma X^2 P(X) = (0^2)(0.18) + (1^2)(0.34) + (2^2)(0.23) + (3^2)(0.21) + (4^2)(0.04) = 3.79$
- $1.59^2 = 2.53$ (rounded to two decimal places).
- $\sigma^2 = 3.79 - 2.53 = 1.26$
- $\sigma = \sqrt{1.26} = 1.12$

6-3 Expectation

The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is

$$\mu = E(X) = \sum X \cdot P(X)$$

The symbol $E(X)$ is used for the expected value.

6-3 Expectation - Example

- A ski resort loses $70,000 per season when it does not snow very much and makes $250,000 when it snows a lot. The probability of it snowing at least 75 inches (i.e., a good season) is 40%. Find the expected profit.

6-3 Expectation - Example

Profit, X	*250,000*	*–70,000*
P(X)	*0.40*	*0.60*

- **The expected profit = ($250,000)(0.40) + (–$70,000)(0.60) = $58,000.**

6-4 The Binomial Distribution

- A binomial experiment is a probability experiment that satisfies the following four requirements:
- Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. Each outcome can be considered as either a success or a failure.

6-4 The Binomial Distribution

- There must be a fixed number of trials.
- The outcomes of each trial must be independent of each other.
- The probability of success must remain the same for each trial.

6-4 The Binomial Distribution

- **The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a binomial distribution.**

6-31

6-4 The Binomial Distribution

- **Notation for the Binomial Distribution:**
- $P(S) = p$, probability of a success
- $P(F) = 1 - p = q$, probability of a failure
- n = number of trials
- X = number of successes.

6-4 Binomial Probability Formula

In a binomial experiment, the probability of exactly X successes in n trials is

$$P(X) = \frac{n!}{(n-X)!\,X!}\, p^X q^{n-X}$$

6-33

6-4 Binomial Probability - Example

- If a student randomly guesses at five multiple-choice questions, find the probability that the student gets exactly three correct. Each question has five possible choices.
- Solution: $n = 5$, $X = 3$, and $p = 1/5$. Then, $P(3) = [5!/((5 - 3)!3!)](1/5)^3(4/5)^2 \approx 0.05$.

6-4 Binomial Probability - Example

- A survey from Teenage Research Unlimited (Northbrook, Illinois.) found that 30% of teenage consumers received their spending money from part-time jobs. If five teenagers are selected at random, find the probability that at least three of them will have part-time jobs.

6-4 Binomial Probability - Example

- **Solution:** $n = 5$, $X = 3, 4$, and 5, and $p = 0.3$.
  Then, $P(X \geq 3) = P(3) + P(4) + P(5) = 0.1323 + 0.0284 + 0.0024 = 0.1631$.
- **NOTE:** You can use Table B in the textbook to find the Binomial probabilities as well.

6-4 Binomial Probability - Example

- A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur on weekend nights involve an intoxicated driver. If a sample of 15 single-vehicle traffic fatalities that occurred on a weekend night is selected, find the probability that exactly 12 involve a driver who is intoxicated.

6-37

6-4 Binomial Probability - Example

- **Solution:** $n = 15$, $X = 12$, and $p = 0.7$. From Table B, $P(X = 12) = 0.170$

6-38

6-4 Mean, Variance, Standard Deviation for the Binomial Distribution - Example

- **A coin is tossed four times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.**
- **Solution:** $n = 4$, $p = 1/2$, and $q = 1/2$.
- $\mu = n \cdot p = (4)(1/2) = 2.$
- $\sigma^2 = n \cdot p \cdot q = (4)(1/2)(1/2) = 1.$
- $\sigma = \sqrt{1} = 1.$