Chapter 9

Hypothesis Testing

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- 9-1 Introduction
- 9-2 Steps in Hypothesis Testing
- 9-3 Large Sample Mean Test
- 9-4 Small Sample Mean Test
- 9-5 Proportion Test

Outline

- 9-6 Variance or Standard Deviation Test
- 9-7 Confidence Intervals and Hypothesis Testing

Objectives

- Understand the definitions used in hypothesis testing.
- State the null and alternative hypotheses.
- Find critical values for the *z* test.

Objectives

- State the five steps used in hypothesis testing.
- Test means for large samples using the *z* test.
- Test means for small samples using the *t* test.

Objectives

- Test proportions using the *z* test.
- Test variances or standard deviation using the chi-square test.
- Test hypotheses using confidence intervals.

9-2 Steps in Hypothesis Testing

- A Statistical hypothesis is a conjecture about a population parameter. This conjecture may or may not be true.
- The null hypothesis, symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value or that there is no difference between two parameters.

9-2 Steps in Hypothesis Testing

- The **alternative hypothesis**, symbolized by H_1, is a statistical hypothesis that states a specific difference between a parameter and a specific value or states that there is a difference between two parameters.

9-2 Steps in Hypothesis Testing - Example

- A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication.

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9-2 Steps in Hypothesis Testing - Example

- What are the hypotheses to test whether the pulse rate will be different from the mean pulse rate of 82 beats per minute?
- H_0: $\mu = 82$ H_1: $\mu \neq 82$
- This is a two-tailed test.

9-2 Steps in Hypothesis Testing - Example

- A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the battery is 36 months, then his hypotheses are
- H_0: $\mu \leq 36$ H_1: $\mu > 36$
- This is a right-tailed test.

9-2 Steps in Hypothesis Testing - Example

- **A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is \$78, her hypotheses about heating costs will be**
- H_0: $\mu \geq \$78$ H_0: $\mu < \$78$
- **This is a left-tailed test.**

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9-2 Steps in Hypothesis Testing

- A **statistical test** uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.

9-2 Steps in Hypothesis Testing

- The numerical value obtained from a statistical test is called the test value.
- In the hypothesis-testing situation, there are four possible outcomes.

9-2 Steps in Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

9-2 Steps in Hypothesis Testing

	H_0 True	H_0 False
Reject H_0	Error Type I	Correct decision
Do not reject H_0	Correct decision	Error Type II

9-2 Steps in Hypothesis Testing

- A type I error occurs if one rejects the null hypothesis when it is true.
- A type II error occurs if one does not reject the null hypothesis when it is false.

9-2 Steps in Hypothesis Testing

- The level of significance is the maximum probability of committing a type I error. This probability is symbolized by α (Greek letter alpha). That is, $P(\text{type I error})=\alpha$.
- $P(\text{type II error}) = \beta$ (Greek letter beta).

9-2 Steps in Hypothesis Testing

- Typical significance levels are: 0.10, 0.05, and 0.01.
- For example, when $\alpha = 0.10$, there is a 10% chance of rejecting a true null hypothesis.

9-2 Steps in Hypothesis Testing

- The critical value(s) separates the critical region from the noncritical region.
- The symbol for critical value is C.V.

9-2 Steps in Hypothesis Testing

- The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.

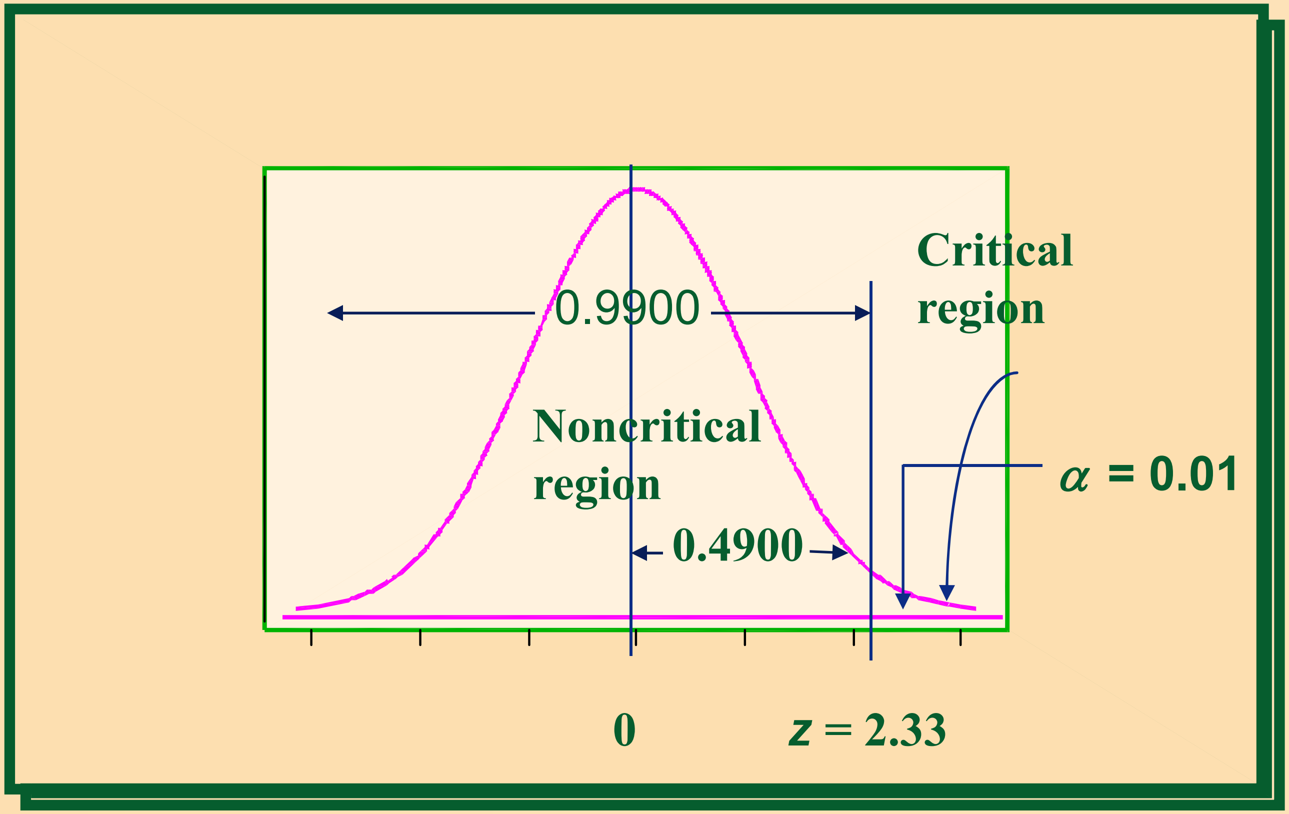
9-2 Steps in Hypothesis Testing

- The noncritical or nonrejection region is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

9-2 Steps in Hypothesis Testing

- A **one-tailed test (right or left)** indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean.

9-2 Finding the Critical Value for $\alpha = 0.01$ (Right-Tailed Test)



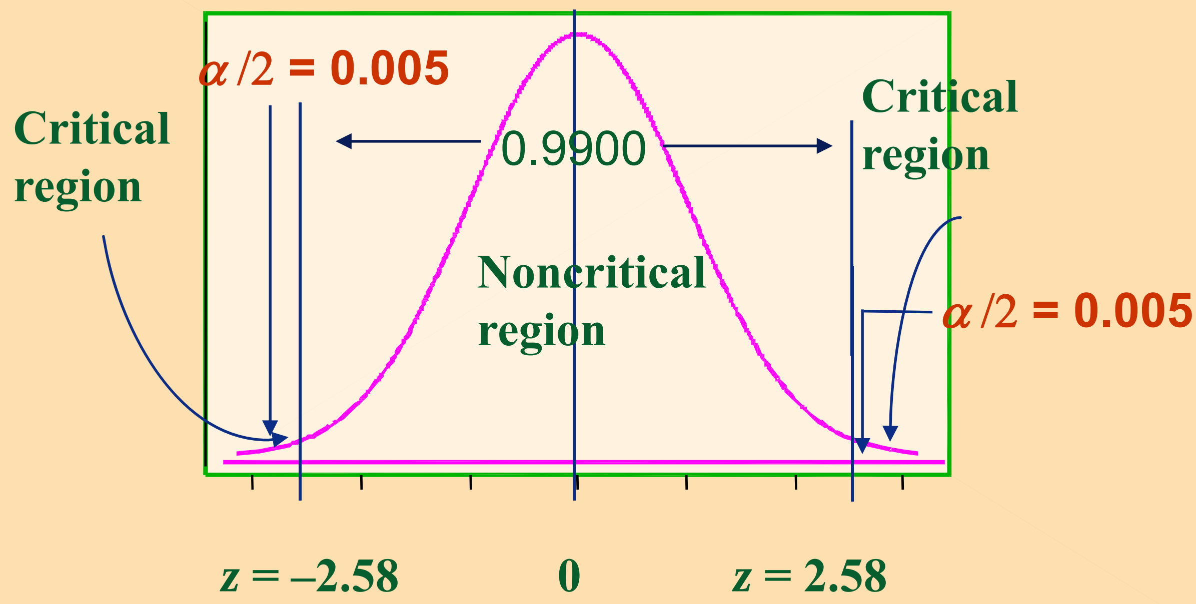
9-2 Finding the Critical Value for $\alpha = 0.01$ (Left-Tailed Test)

- For a left-tailed test when $\alpha = 0.01$, the critical value will be –2.33 and the critical region will be to the left of –2.33.

9-2 Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)

- In a two-tailed test, the null hypothesis should be rejected when the test value is in either of the two critical regions.

9-2 Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)



9-3 Large Sample Mean Test

- The **z test** is a statistical test for the mean of a population. It can be used when $n \geq 30$, or when the population is normally distributed and σ is known.
- The formula for the z test is given on the next slide.

9-3 Large Sample Mean Test

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$ = *sample mean*

μ = *hypothesized population mean*

σ = *population deviation*

n = *sample size*

9-3 Large Sample Mean Test - Example

- **A researcher reports that the average salary of assistant professors is more than \$42,000. A sample of 30 assistant professors has a mean salary of \$43,260. At α = 0.05, test the claim that assistant professors earn more than \$42,000 a year. The standard deviation of the population is \$5230.**

9-3 Large Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu \leq$ \$42,000 H_1: $\mu >$ \$42,000 (claim)
- **Step 2:** Find the critical value. Since α = 0.05 and the test is a right-tailed test, the critical value is z = +1.65.
- **Step 3:** Compute the test value.

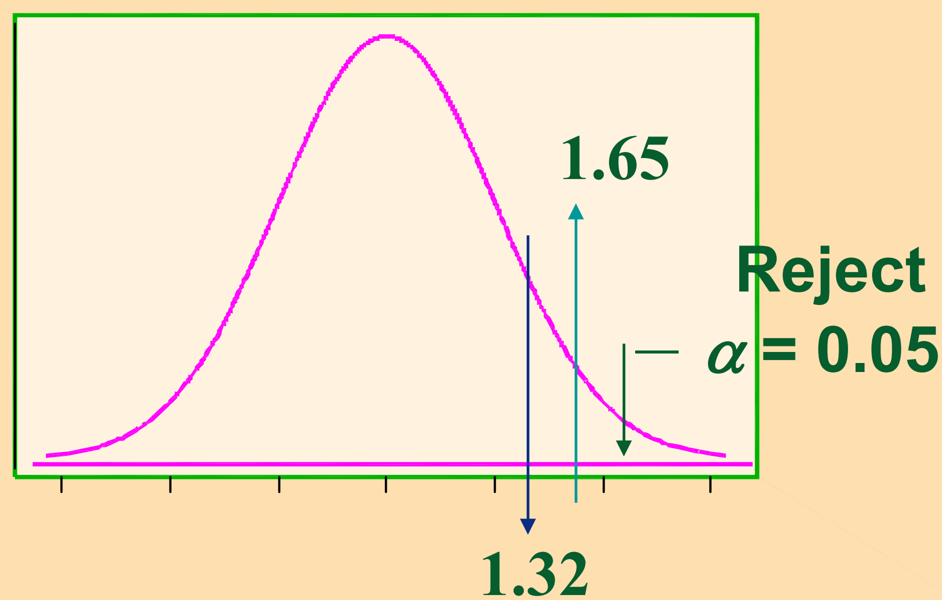
9-3 Large Sample Mean Test - Example

- Step 3: $z = [43{,}260 - 42{,}000]/[5230/\sqrt{30}] = 1.32$.
- Step 4: Make the decision. Since the test value, +1.32, is less than the critical value, +1.65, and not in the critical region, the decision is "Do not reject the null hypothesis."

9-3 Large Sample Mean Test - Example

- **Step 5:** Summarize the results. There is not enough evidence to support the claim that assistant professors earn more on average than $42,000 a year.
- See the next slide for the figure.

9-3 Large Sample Mean Test - Example



9-3 Large Sample Mean Test - Example

- A national magazine claims that the average college student watches less television than the general public. The national average is 29.4 hours per week, with a standard deviation of 2 hours. A sample of 30 college students has a mean of 27 hours. Is there enough evidence to support the claim at $\alpha = 0.01$?

9-3 Large Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu \geq 29.4$ H_1: $\mu < 29.4$ (claim)
- **Step 2:** Find the critical value. Since $\alpha = 0.01$ and the test is a left-tailed test, the critical value is $z = -2.33$.
- **Step 3:** Compute the test value.

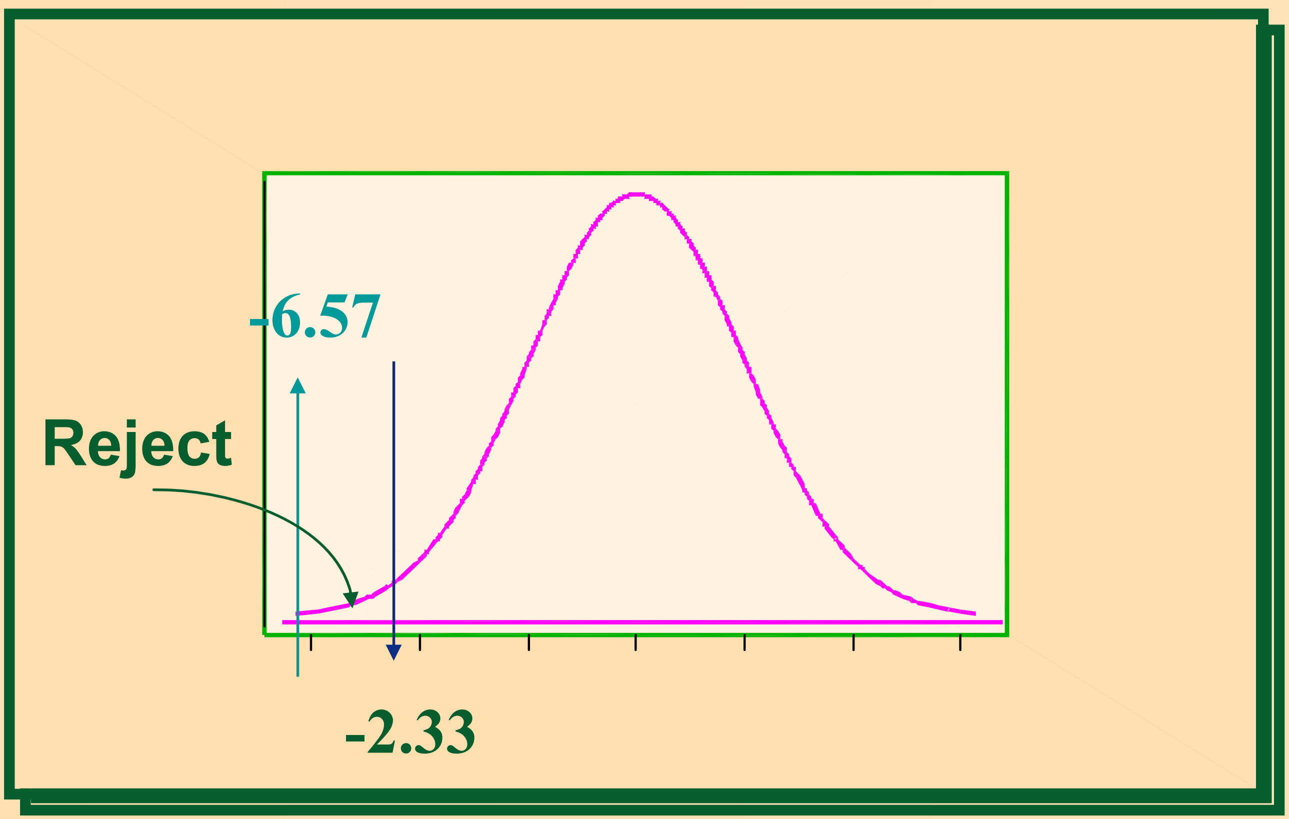
9-3 Large Sample Mean Test - Example

- **Step 3:** $z = [27 - 29.4]/[2/\sqrt{30}] = -6.57$.
- **Step 4:** Make the decision. Since the test value, -6.57, falls in the critical region, the decision is to reject the null hypothesis.

9-3 Large Sample Mean Test - Example

- **Step 5:** **Summarize the results. There is enough evidence to support the claim that college students watch less television than the general public.**
- **See the next slide for the figure.**

9-3 Large Sample Mean Test - Example



9-3 Large Sample Mean Test - Example

- The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a large hospital, a researcher selected a random sample of 35 stroke victims and found that the average cost of their rehabilitation is $25,226.

9-3 Large Sample Mean Test - Example

- The standard deviation of the population is \$3,251. At $\alpha = 0.01$, can it be concluded that the average cost at a large hospital is different from \$24,672?
- Step 1: State the hypotheses and identify the claim.
- H_0: $\mu = \$24,672$ H_1: $\mu \neq \$24,672$ (claim)

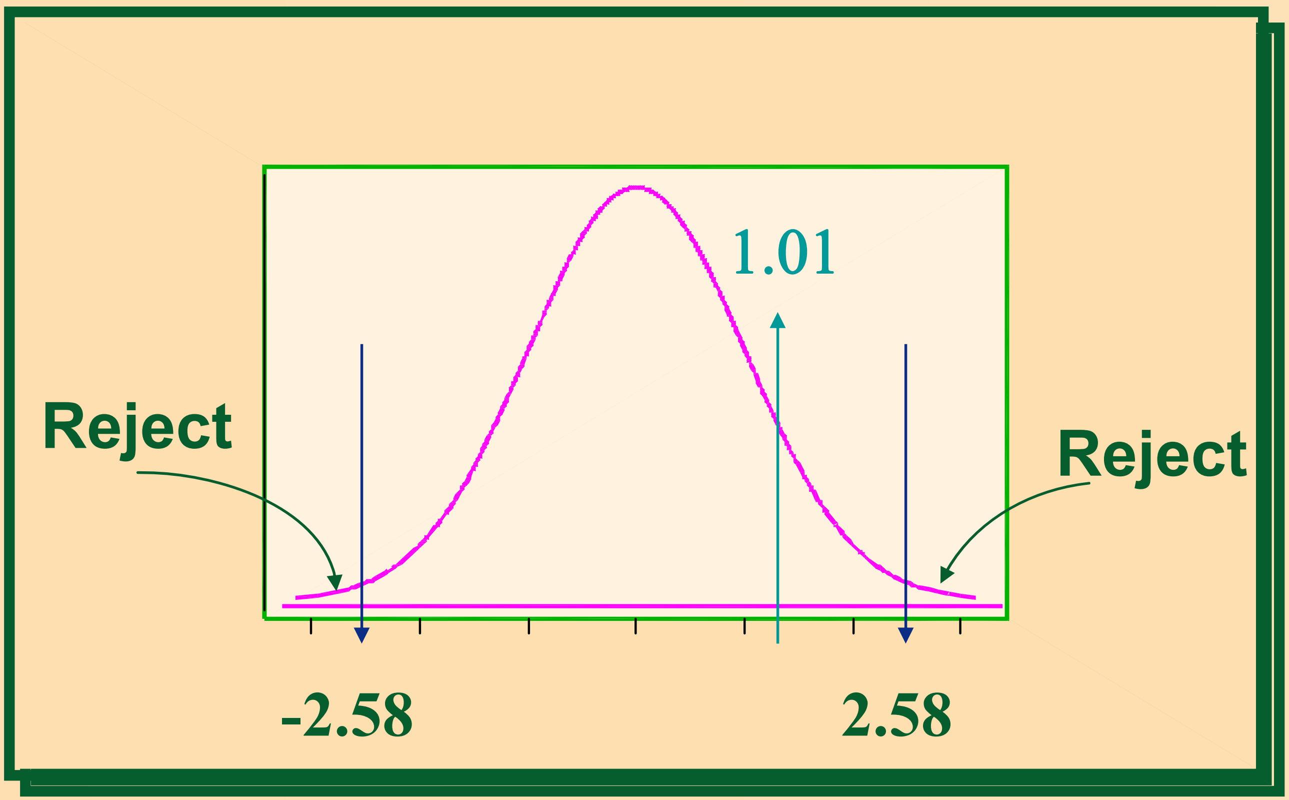
9-3 Large Sample Mean Test - Example

- **Step 2:** Find the critical values. Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are $z = -2.58$ and $+2.58$.
- **Step 3:** Compute the test value.
- **Step 3:** $z = [25{,}226 - 24{,}672]/[3{,}251/\sqrt{35}] = 1.01$.

9-3 Large Sample Mean Test - Example

- **Step 4:** Make the decision. Do not reject the null hypothesis, since the test value falls in the noncritical region.
- **Step 5:** Summarize the results. There is not enough evidence to support the claim that the average cost of rehabilitation at the large hospital is different from $24,672.

9-3 Large Sample Mean Test - Example



9-3 *P*-Values

- **Besides listing an α value, many computer statistical packages give a *P*-value for hypothesis tests. The *P*-value is the actual probability of getting the sample mean value or a more extreme sample mean value in the direction of the alternative hypothesis (> or <) if the null hypothesis is true.**

9-3 *P*-Values

- The *P*-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample mean or a more extreme sample mean occurring if the null hypothesis is true.

9-3 *P*-Values - Example

- A researcher wishes to test the claim that the average age of lifeguards in Ocean City is greater than 24 years. She selects a sample of 36 guards and finds the mean of the sample to be 24.7 years, with a standard deviation of 2 years. Is there evidence to support the claim at $\alpha = 0.05$? Find the *P*-value.

9-3 *P*-Values - Example

- **Step 1: State the hypotheses and identify the claim.**
- H_0: $\mu \leq 24$ H_1: $\mu > 24$ (claim)
- **Step 2: Compute the test value.**

$$z = \frac{24.7 - 24}{2/\sqrt{36}} = 2.10$$

9-3 *P*-Values - Example

- **Step 3:** Using Table E in Appendix C, find the corresponding area under the normal distribution for $z = 2.10$. It is 0.4821
- **Step 4:** Subtract this value for the area from 0.5000 to find the area in the right tail.

  $$0.5000 - 0.4821 = 0.0179$$

  Hence the *P*-value is 0.0179.

9-3 *P*-Values - Example

- Step 5: Make the decision. Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.
- Step 6: Summarize the results. There is enough evidence to support the claim that the average age of lifeguards in Ocean City is greater than 24 years.

9-3 *P*-Values - Example

- A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the sample is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

9-3 *P*-Values - Example

- **Step 1: State the hypotheses and identify the claim.**
- H_0: $\mu = 8$ (claim) H_1: $\mu \neq 8$
- **Step 2: Compute the test value.**

$$z = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89$$

9-3 *P*-Values - Example

- **Step 3:** **Using table E, find the corresponding area for z = 1.89. It is 0.4706.**
- **Step 4:** **Subtract the value from 0.5000.**

  0.5000 – 0.4706 = 0.0294

9-3 *P*-Values - Example

- **Step 5:** Make the decision: Since this test is two-tailed, the value 0.0294 must be doubled; 2(0.0294) = 0.0588. Hence, the decision is not to reject the null hypothesis, since the *P*-value is greater than 0.05.
- **Step 6:** Summarize the results. There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.

9-4 Small Sample Mean Test

- When the population standard deviation is unknown and $n < 30$, the z test is inappropriate for testing hypotheses involving means.
- The *t* **test** is used in this case.
- Properties for the t distribution are given in Chapter 8.

9-4 Small Sample Mean Test - Formula for *t* test

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

where

$\overline{X}$ = *sample mean*

μ = *hypothesized population mean*

s = *sample standard deviation*

n = *sample size*

degrees of freedom $= n - 1$

9-4 Small Sample Mean Test - Example

- A job placement director claims that the average starting salary for nurses is \$24,000. A sample of 10 nurses has a mean of \$23,450 and a standard deviation of \$400. Is there enough evidence to reject the director's claim at $\alpha = 0.05$?

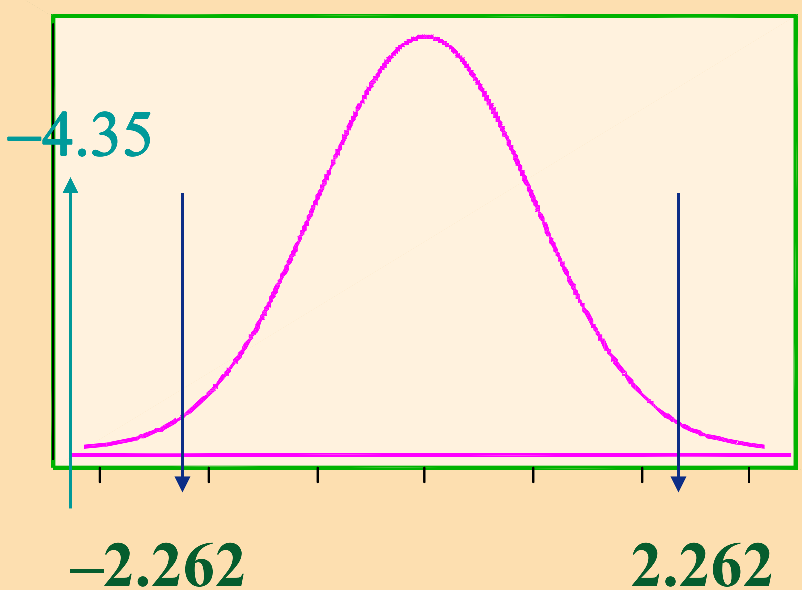
9-4 Small Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu = \$24{,}000$ (claim) H_1: $\mu \neq \$24{,}000$
- **Step 2:** Find the critical value. Since $\alpha = 0.05$ and the test is a two-tailed test, the critical values are $t = -2.262$ and $+2.262$ with d.f. = 9.

9-4 Small Sample Mean Test - Example

- **Step 3:** **Compute the test value.**
  $t = [23{,}450 - 24{,}000]/[400/\sqrt{10}] = -4.35.$
- **Step 4:** **Reject the null hypothesis, since $-4.35 < -2.262$.**
- **Step 5:** **There is enough evidence to reject the claim that the starting salary of nurses is $24,000.**

9-3 Small Sample Mean Test - Example



9-5 Proportion Test

- Since the normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.
- The formula for the *z* test for proportions is given on the next slide.

9-5 Formula for the z Test for Proportions

$$z = \frac{X - \mu}{\sigma} \quad or \quad z = \frac{X - np}{\sqrt{npq}}$$

where

$$\mu = np$$

$$\sigma = \sqrt{npq}$$

9-5 Proportion Test - Example

- An educator estimates that the dropout rate for seniors at high schools in Ohio is 15%. Last year, 38 seniors from a random sample of 200 Ohio seniors withdrew. At $\alpha = 0.05$, is there enough evidence to reject the educator's claim?

9-5 Proportion Test - Example

- **Step 1:** **State the hypotheses and identify the claim.**
- H_0: $p = 0.15$ (claim) H_1: $p \neq 0.15$
- **Step 2:** **Find the mean and standard deviation.** $\mu = np = (200)(0.15) = 30$ **and** $\sigma = \sqrt{(200)(0.15)(0.85)} = 5.05$.
- **Step 3:** **Find the critical values. Since** $\alpha = 0.05$ **and the test is two-tailed the critical values are** $z = \pm 1.96$.

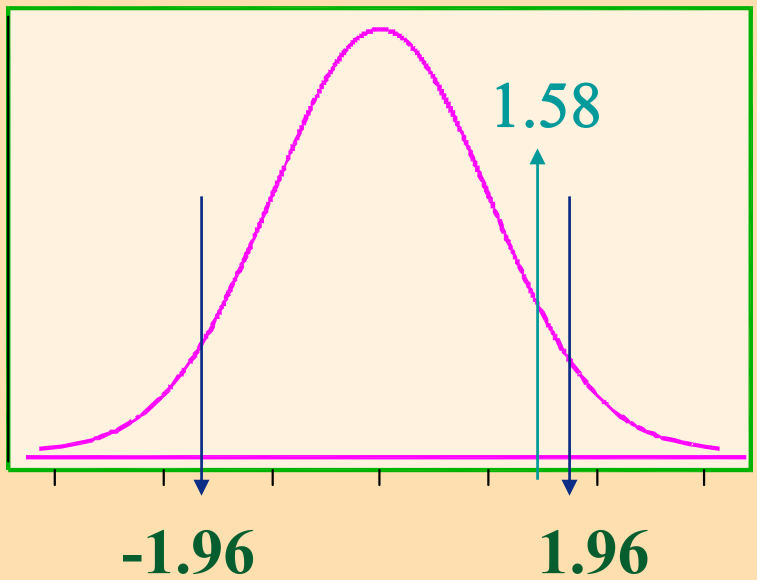
9-5 Proportion Test - Example

- **Step 4:** Compute the test value.
  $z = [38 - 30]/[5.05] = 1.58$.
- **Step 5:** Do not reject the null hypothesis, since the test value falls outside the critical region.

9-5 Proportion Test - Example

- **Step 6:** **Summarize the results. There is not enough evidence to reject the claim that the dropout rate for seniors in high schools in Ohio is 15%.**
- **Note: For one-tailed test for proportions, follow procedures for the large sample mean test.**

9-5 Proportion Test - Example



9-6 Variance or Standard Deviation Test - Formula

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \textit{with}\ \mathrm{d.f.} = n-1$$

where

$n = \textit{sample size}$

$\sigma^2 = \textit{population variance}$

$s^2 = \textit{sample variance}$

9-6 Assumptions for the Chi-Square Test for a Single Variance

- The sample must be randomly selected from the population.
- The population must be normally distributed for the variable under study.
- The observations must be independent of each other.

9-6 Variance Test - Example

- An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 225$) at $\alpha = 0.05$?

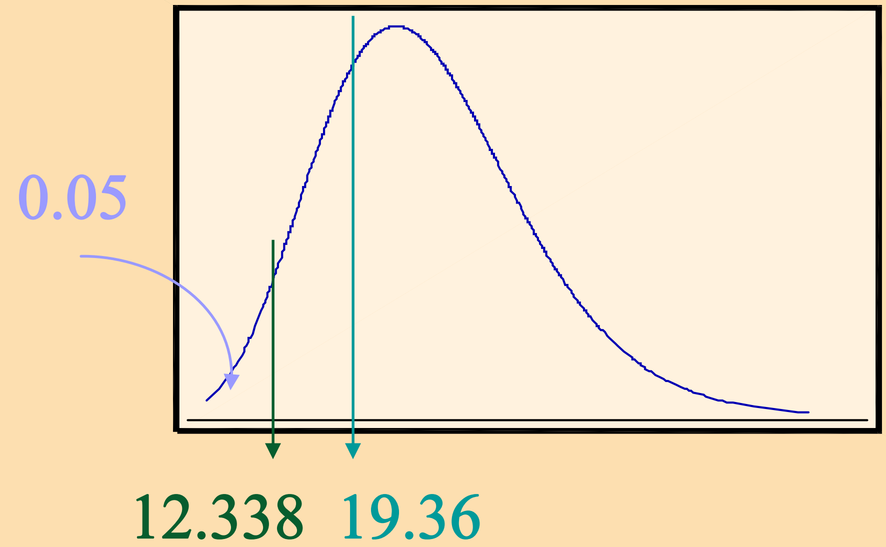
9-6 Variance Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\sigma^2 \geq$ 225 H_1: $\sigma^2 <$ 225 (claim)
- **Step 2:** Find the critical value. Since this test is left-tailed and α = 0.05, use the value 1 – 0.05 = 0.95.
  The d.f. = 23 – 1 = 22. Hence, the critical value is 12.338.

9-6 Variance Test - Example

- **Step 3:** Compute the test value.
  $\chi^2 = (23 - 1)(198)/225 = 19.36.$
- **Step 4:** Make a decision. Do not reject the null hypothesis, since the test value falls outside the the critical region.

9-6 Variance Test - Example



9-6 Variance Test

- **Note:** When the test is two-tailed, you will need to find χ^2(left) and χ^2(right) and check whether the test value is less than χ^2(left) or whether it is greater than χ^2(right) in order to reject the null hypothesis.

9-7 Confidence Intervals and Hypothesis Testing - Example

- **Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated, at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.**

9-7 Confidence Intervals and Hypothesis Testing - Example

- H_0: $\mu = 5$ H_1: $\mu \neq 5$ (claim)
- The critical values are +1.96 and – 1.96
- The test value is $z = \frac{4.6 - 5.00}{0.7/\sqrt{50}} = -4.04$
- Since – 4.04 < –1.96, the null hypothesis is rejected.
- There is enough evidence to support the claim that the bags do not weigh 5 pounds.

9-7 Confidence Intervals and Hypothesis Testing - Example

- **The 95% confidence interval for the mean is given by**

$$\overline{X} - z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \overline{X} + z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$4.6 - (1.96)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6 + (1.96)\left(\frac{0.7}{\sqrt{50}}\right)$$

$$4.406 < \mu < 4.794$$

- **Notice that the 95% confidence interval of μ does not contain the hypothesized value μ = 5. Hence, there is agreement between the hypothesis test and the confidence interval.**